A Proposal for the Worldvolume Action of Multiple M5-Branes

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seminar at NTNU, 9/5/2013
qbased on

1. A Theory of Non-Abelian Tensor Gauge Field with Non-Abelian Gauge Symmetry G x G, Chong-Sun Chu, arXiv:1108.5131.
2. Non-abelian Action for Multiple Five-Branes with Self-Dual Tensors, Chong-Sun Chu, Sheng-Lan Ko, arXiv:1203.4224.
3. Non-Abelian Self-Dual String Solutions, Chong-Sun Chu, Sheng-Lan Ko, Pichet Vanichchapongjaroen, arXiv:1207.1095.
4. Non-abelian Self-Dual String and M2-M5 Branes Intersection in Supergravity Chong-Sun Chu, Pichet Vanichchapongjaroen, arXiv:1304.4322.

and unpublished results.

Outline

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Mysteries of M5-branes

What we know:

- The low energy worldvolume dynamics is given by a 6d (2,0) SCFT with $SO(5)$ R-symmetry.

  (Strominger, Witten)

  The (2,0) tensor multiples contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions.

  (Gibbons, Townsend; Strominger; Kaplan, Michelson)

What we don't know:

- What is the form of the gauge symmetry for multiple M5-branes ?
- Interacting self-dual dynamics on M5-branes worldvolume?

Self-dual dynamics for multiple M5-branes (?)

- Generally, it is well known to be difficult to write down a Lorentz invariant action for self-dual dynamics.

  (Siegel 84; Floreanini, Jackiw 87)

- For a single M5 case, problem solved by sacrificing manifest 6d Lorentz symmetry.

  (Henneaux-Teitelboim 88; Perry-Schwarz 97)

  The action was later generalized to include kappa symmetry

  (Aganagic, Park, Popescu, Schwarz, 97)

  Covariant construction was given by PST

  (Pasti-Sorokin-Tonin)

- Not clear how to do this for $N > 1$ due to the other problem that an appropriate generalization of the tensor gauge symmetry was not known.

Enhanced gauge symmetry of multiple M5-branes (?)

- For multiple D-branes, symmetry is enhanced from $U(1)$ to $U(N)$:

$$\delta A^a_\mu = \partial_\mu \Lambda^a + [A_\mu, \Lambda]^a, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + [A_\mu, A_\nu]^a.$$

- For multiple M5-branes, it is not known how to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B^a_{\mu\nu} = \partial_\mu \Lambda^a_\nu - \partial_\nu \Lambda^a_\mu + (?), \quad H^a_{\mu\nu\lambda} = \partial_\mu B^a_{\nu\lambda} + \partial_\nu B^a_{\lambda\mu} + \partial_\lambda B^a_{\mu\nu} + (?).$$

  to have nontrivial self interaction.

- Moreover, exists no-go theorems: there is no nontrivial deformation of the Abelian 2-form gauge theory if *locality* of the action and the transformation laws are assumed.

  (Henneaux; Bekaert; Sevrin; Nepomechie)

- These no-go theorems, however, suggest an important direction of given up locality.

- The need of nonlocality for M5-branes should not be surprising: ABJM and BLG theory for multiple M2-branes are also non-local.
- We will get around the no-go theorem by similarly introducing a set of auxillary fields: the theory become nonlocal when these fields are eliminated using their equations of motion.

- In this talk, I will explain a 6d proposal of the low energy worldvolume theory (Ko and Chu).
- There is also a 5d proposal of the (2,0) theory as strong coupling limit of 5d SYM. (Douglas; Lambert, Papageorgakis and Schmidt-Sommerfeld)
- analogy:
  low energy SUGRA $\leftarrow$ quantum mechanical M-theory (proposed to be defined by D0s' SQM)
  low energy theory of M5-branes $\leftarrow$ quantum M5-branes theory (proposed to be defined by 5d SYM)

Other related works/approaches: Smith, Ho, Huang, Masuo, Samtleben, Sezgin, Wimmer, Wulff, Saemann, Wolf, Czech, Huang, Rozali, Tachikawa, H.C. Kim, S. Kim, Koh, K. Lee, S. Lee, Bolognesi, Maxfield, Sethi, Bak, Gustavsson, Hosomichi, Seong, Nosaka, Terashima, Kallen, Minahan, Nedelin, Zabzine, Palmer, Sorokin, Bandos, ...

Outline

Review of the Perry-Schwarz formulation

- Perry-Schwarz formulation:
  1. A direction, say x_5, is treated differently: denote the 5d and 6d coord. by x^μ and $x^M = (x^\mu, x^5)$.
  2. the tensor gauge field potential is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. It can be thought of as a (tensor) gauge fixed formulation in which $B_{\mu 5}$ never appear.
- Perry and Schwarz considered the action:

$$S_0(B) = \frac{1}{2} \int d^6x \left(-\tilde{H}^{\mu\nu} \tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu} \partial_5 B_{\mu\nu} \right)$$

where

$$\tilde{H}^{\mu\nu} := \frac{1}{6} \epsilon^{\mu\nu\rho\lambda\sigma} H_{\rho\lambda\sigma}.$$

Note that manifest Lorentz symmetry is lost.

- EOM:

$$\epsilon^{\mu\nu\rho\lambda\sigma}\partial_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu, \quad \text{for arbitrary } \alpha_\mu.$$

- The action is invariant under the gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \quad \text{for arbitrary } \varphi_\mu.$$

This allows one to reduce the general solution to the EOM to the first order form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

This is the self-duality equation in this theory.

Modified Lorentz symmetry

- The action has manifest 5d Lorentz invariance and a non-manifest Lorentz symmetry mixing μ with the 5 direction.

Lorentz transformation (active view)

Standard Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot L) B_{\mu\nu} + \delta_{\rm spin} B_{\mu\nu}$$

has a orbital part:

$$\Lambda \cdot L = (\Lambda \cdot x)\partial_5 - x_5 (\Lambda \cdot \partial)$$

and a spin part:

$$\delta_{\rm spin} B_{\mu\nu} = \Lambda_\nu B_{\mu 5} - \Lambda_\mu B_{\nu 5}.$$

- PS proposed to consider the following transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - x_5 (\Lambda \cdot \partial) B_{\mu\nu},$$

where $\Lambda_\mu = \Lambda_{5\mu}$ denote the corresponding infinitesimal transformation parameters.

1. On shell, it is equal to the standard Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu} = (\Lambda \cdot x)\partial_5 B_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu}$$

2. Commutator:

$$[\delta_{\Lambda_1}, \delta_{\Lambda_2}]B_{\mu\nu} = \delta^{(5d)}_{\Lambda_{\alpha\beta}} B_{\mu\nu} + \text{EOM} + \text{gauge symmetry}$$

where

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \qquad \varphi_\nu = x^\alpha \Lambda_{\alpha\lambda} B_\nu{}^\lambda$$

is a gauge symmetry of the PS theory. So the PS transformation can be considered as a (modified) Lorentz transformation.

Note that modified Lorentz symmetry is typical of action of self-dual dynamics

(Siegel 84)

Non-abelian action

The idea is to try to generalize the Perry-Schwarz approach:

1. Represent the self-dual tensor gauge field by a 5×5 antisymmetric field $B^a_{\mu\nu}$ in the adjoint and giving up manifest 6d Lorentz symmetry
2. Moreover we'll introduce a set of YM gauge fields A^a_μ for gauge group G.

(Chu, Smith; Chu)

The Action

Our proposed action $S = S_0 + S_E$ consists of two pieces:

- $$S_0 = \frac{1}{2}\int d^6x \,\mathrm{tr}\left(-\tilde{H}^{\mu\nu}\tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu}\partial_5 B_{\mu\nu}\right),$$

  is the non-abelian generalization of the Perry-Schwarz action, where

  $$H_{\mu\nu\lambda} = D_{[\mu}B_{\nu\lambda]}, \quad D_\mu = \partial_\mu + A_\mu.$$

- $$S_E = \int d^5x \,\mathrm{tr}\left((F_{\mu\nu} - c\int dx_5\, \tilde{H}_{\mu\nu})E^{\mu\nu}\right).$$

  where $E_{\mu\nu}$ is a 5d auxiliary field implementing the constraint

  $$F_{\mu\nu} = c\int dx_5\, \tilde{H}_{\mu\nu}$$

- Note that there is no $B_{\mu 5}$ and A_5. A_μ and $E_{\mu\nu}$ live in 5-dimensions
- Note also the presence of fields $A_\mu, E_{\mu\nu}$ that is not expected from (2,0) susy.

The action is invariant under:

1. Yang-Mills gauge symmetry

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \delta B_{\mu\nu} = [B_{\mu\nu}, \Lambda], \quad \delta E_{\mu\nu} = [E_{\mu\nu}, \Lambda].$$

2. Tensor gauge symmetry:

$$\delta_T A_\mu = 0, \quad \delta_T B_{\mu\nu} = D_{[\mu}\Lambda_{\nu]}, \quad \delta_T E_{\mu\nu} = 0,$$

   for arbitrary $\Lambda_\mu(x^M)$ such that $[F_{[\mu\nu}, \Lambda_{\lambda]}] = 0$.

3. Moreover there is a gauge symmetry

$$\delta E_{\mu\nu} = \alpha_{\mu\nu}$$

   for arbitrary $\alpha(x^\lambda)$ such that $D_{[\mu}\alpha_{\nu\lambda]} = 0, \quad D^\mu \alpha_{\mu\lambda} = 0.$

Property 1: Self-Duality

- EOM of $E_{\mu\nu}$ gives the constraint

$$F_{\mu\nu} = c \int dx_5 \, \tilde{H}_{\mu\nu}.$$

- EOM of $B_{\mu\nu}$

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho (\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

  has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \Phi_{\lambda\sigma},$$

  where $D_{[\lambda}\Phi_{\mu\nu]} = 0$.

- Again with an appropriate fixing of the tensor gauge symmetry, one can reduce the second order EOM to the self-duality equation

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

Property 2: Degrees of freedom

- After eliminating the aux field $E_{\mu\nu}$ and substituting the $F = H$ constraint, the resulting action becomes highly nonlinear and interacting. To count the degrees of freedom, we use the linearized theory.

  At the quadratic level, the non-abelian action is simply given by $\dim G$ copies of the Perry-Schwarz action. We obtain $3 \times \dim G$ degrees of freedom in $B_{\mu\nu}$.

- Our theory contains $3 \times \dim G$ degrees of freedom as required by (2,0) supersymmetry.

Property 3: Lorentz Symmetry

- The action is invariant under the 5-μ Lorentz transformation:

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} + x_5 \Lambda^\kappa H_{\kappa\mu\nu} + \Lambda^\kappa \phi_{\mu\nu\kappa},$$

where

$$\phi^a_{\mu\nu\kappa} = \int dy\ G^{ab\mu'\nu'}_{\mu\nu}(x,y) J^b_{\mu'\nu'\kappa}(y)$$

$J_{\mu\nu\kappa}$ is some expression linear in H
and $G^{ab}_{\mu\nu,\mu'\nu'}(x,y)$ is the Green function satisfying

$$\partial_5 G^{ab\mu'\nu'}_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta\gamma}(D^{(y)}_\alpha)^a{}_c G^{cb\mu'\nu'}_{\beta\gamma} = \delta^{\mu'\nu'}_{\mu\nu}\delta^{ab}\delta^{(6)}(x-y)$$

with the BC: $G^{ab\mu'\nu'}_{\mu\nu}(x,y) = 0, \quad |x_5| \to \infty.$

- As before, the commutator closes to the standard 5d Lorentz transformation plus terms vanishing onshell, plus a gauge transformation.

- Note that our proposed Lorentz symmetry is nonlocal.

  This is not unexpected since we are working in a gauge fixed formulation without $B_{\mu 5}$. The Lorentz symmetry is also nonlocal for QED in Coulomb gauge or string in lightcone gauge.

Property 4: Reduction to D4-branes

- Consider a compactification of x_5 on a circle of radius R. The dimensional reduced action reads

$$S = \frac{2\pi R}{2} \int d^5x \, \mathrm{tr} \left(-\tilde{H}_{\mu\nu}^2 + (F_{\mu\nu} - 2\pi R c \tilde{H}_{\mu\nu}) E^{\mu\nu} \right)$$

- Integrate out $E_{\mu\nu}$, we obtain

$$F_{\mu\nu} = 2\pi R c \tilde{H}_{\mu\nu}.$$

and eliminate $\tilde{H}_{\mu\nu}$, we obtain the 5d Yang-Mills action

$$S_{YM} = -\frac{1}{4\pi R c^2} \int d^5x \, \mathrm{tr} \, F_{\mu\nu}^2.$$

- This gives the YM coupling and the gauge group to be

$$g_{YM}^2 = R c^2, \quad G = U(N)$$

for a system of N M5-branes.

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- EOM gives $D^\mu F_{\mu\nu} = 0$ instead of

$$D_\mu F^{\mu\nu} = -\frac{\pi R}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]?$$

- Need to be more careful with the implementation of Delta function:

$$\int [DA][DB][DE]e^{-S} = \int [DA][DB]e^{-S_{YM}}\delta(F_{\mu\nu} - 2\pi R\tilde{H}_{\mu\nu}) = \int [DA]e^{-S_{YM}-S'},$$

where consistency requires that

$$\frac{\delta S'}{\delta A_\nu} = \frac{1}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]$$

- The 5d theory is thus given by the action $S_{5d} = S_{YM} + S'$.
  S' describes a higher derivative correction term to the Yang-Mills theory since $[F, B] \sim DDB$ and B is of the order of F.
  It might be possible that S' captures the non-abelian DBI action of D4-branes.

- Summarizing, we have constructed a non-abelian action of tensor fields with the properties:
  1. the action admits a self-duality equation of motion,
  2. the action has manifest 5d Lorentz symmetry and a modified 6d Lorentz symmetry,
  3. on dimensional reduction, the action gives the 5d Yang-Mills action plus corrections.

  Based on these properties, we propose our action to be the bosonic theory describing the gauge sector of coincident M5-branes in flat space.

To do:

- We still need to include the scalar field and fermions, and to susy complete the action.
- Another interesting direction is to explore the dynamical content of the theory.

Outline

Self-dual string on M5-brane

- M2-branes can end on M5-brane. The endpoint gives strings living on the M5-brane.
  These self-dual strings appear as solitons of the M5-brane theory.
- In the Abelian case, the self-dual string solition has been obtained by Perry-Schwarz (1996) and also by Howe-Lambert-West (1997).

- For the non-abelian theory, the equations of motion to be solved are:

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

$$F_{\mu\nu} = c \int dx_5 \, \tilde{H}_{\mu\nu}$$

Need to do:

1. The gauge field is auxiliary:

$$F_{\mu\nu} = c \int dx_5 \, \tilde{H}_{\mu\nu} = c(B_{\mu\nu}(x_5 = \infty) - B_{\mu\nu}(x_5 = -\infty))$$

   Therefore if the non-Abelian solution is translationally invariant along x^5, then $F_{\mu\nu} = 0$ is trivial.
   construct a string aligns in a different direction, say x^4.

2. supersymmetrize to get a BPS solution.

- For the abelian case, Perry-Schwarz has obtained a self-dual string solution:

$$B_{ij} = -\frac{1}{2}\frac{\beta\epsilon_{ijk}x_k}{r^3}\left(\frac{x^5 r}{\rho^2} + \tan^{-1}(x^5/r)\right), \qquad B_{04} = -\frac{\beta}{2\rho^2},$$

$i, j = 1, 2, 3.$

- Although the auxillary field does not appear in the PS construction, it is amazing that

$$F_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijk}x_k}{r^3}, \qquad F_{04} = 0$$

i.e. a Dirac monopole in the (x, y, z) subspace if $c\beta = -2/\pi$!

- It turns out the use of an non-abelian monopole in place of the Dirac monopole is precisely what is needed to construct the non-abelian self-dual string solution.
- Here we have two candidates of non-abelian monopole: the Wu-Yang monopole (non-compact case) and the 't Hooft-Polyakov monopole (compact case).

Non-abelian Wu-Yang & 't Hooft-Polyakov monopole

Wu-Yang

- Consider $SU(2)$ gauge group

$$[T^a, T^b] = i\epsilon^{abc} T^c, \quad a, b, c = 1, 2, 3.$$

- The non-abelian Wu-Yang monopole is given by

$$A_i^a = -\epsilon_{aik} \frac{x_k}{r^2}, \qquad F_{ij}^a = \epsilon_{ijm} \frac{x_m x_a}{r^4}, \qquad i, j = 1, 2, 3.$$

- Note that the field strength for the Wu-Yang solution is related to the field strength of the Dirac monopole by a simple relation:

$$F_{ij}^a = F_{ij}^{(\mathrm{Dirac})} \frac{x^a}{r}.$$

- Note that the color magnetic charge vanishes

$$\int_{S^2} F^a = 0$$

and the Wu-Yang solution is actually not a monopole. Neverthless it plays a key role in the construction of non-abelian monopole of 't Hooft-Polyakov.

't Hooft-Polyakov

- In the BPS limit, the 't Hooft-Polyakov monopole satisfies

$$\frac{1}{2}\epsilon_{ijk}F_{ij} = D_k\phi, \quad D_k^2\phi = 0$$

where ϕ is an adjoint Higgs scalar field.

- The solution is given by

$$A_i^a = -\epsilon_{aik}\frac{x^k}{r^2}(1 - k_v(r)), \qquad \phi^a = \frac{vx^a}{r}h_v(r),$$

where

$$k_v(r) := \frac{vr}{\sinh(vr)}, \qquad h_v(r) := \coth(vr) - \frac{1}{vr}.$$

- Asymptotically $r \to \infty$, we have

$$A_i^a \to -\epsilon_{aik}\frac{x^k}{r^2}, \qquad \phi^a \to \frac{|v|x^a}{r} := \phi_\infty,$$

i.e. precisely the Wu-Yang monopole at infinity.

- Unbroken $U(1)$ gauge symmetry at infinity may be identified as the electromagnetic gauge group and the electromagnetic field strength can be obtained as a projection:

$$\mathcal{F}_{ij} = F^a_{ij}\frac{\phi^a}{|v|} = \epsilon_{ijk}\frac{x^k}{r^3}, \qquad \text{for large } r.$$

The magnetic charge is given by $p = \int_{S^2}\mathcal{F} = 4\pi$, which corresponds to a magnetic monopole of unit charge.

- Note that at the core $r \to 0$, we have

$$A_i \to 0, \qquad \phi \to 0$$

and hence the $SU(2)$ symmetry is restored at the monopole core.

- Although we do not have the full (2,0) supersymmetric theory, one can argue (a simple dimensional analysis) that the self-duality equation of motion is not modified by the presence of the scalar fields.
- As for the scalar field's EOM, the self-interacting potential vanishes if there is only one scalar field turned on (R-symmetry). As a result, the equation of motion of the scalar field is

$$D_M^2 \phi = 0.$$

- A reasonable form of the BPS equation is the non-abelian generalization of the BPS equation of Howe-Lambert-West:

$$H_{ijk} = \epsilon_{ijk} \partial_5 \phi, \qquad H_{ij5} = -\epsilon_{ijk} D_k \phi.$$

- This follows immediately from the supersymmetry transformation

$$\delta \psi = (\Gamma^M \Gamma^I D_M \phi^I + \frac{1}{3!2} \Gamma^{MNP} H_{MNP}) \epsilon$$

and the 1/2 BPS condition

$$\Gamma^{046} \epsilon = -\epsilon.$$

Note: this is the most natural non-abelian generalization of the abelian (2,0) supersymmetry transformation.

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Non-abelian self-dual string solution

- Inspired by the relation of Dirac monopole to the Wu-Yang solution, try the ansatz

$$H^a_{\mu\nu\lambda} = H^{(\mathrm{PS})}_{\mu\nu\lambda}\frac{x^a}{r}$$

Here $r = \sqrt{x^2+y^2+z^2}$ and $H^{(\mathrm{PS})}_{\mu\nu\lambda}$ is the field strength for the linearized Perry-Schwarz solution aligning in the x^4 direction. Self-duality is automatically satisfied!

- B can be obtained by integrating $H_{\mu\nu5} = \partial_5 B_{\mu\nu}$ and we obtain

$$B^a_{\mu\nu} = B^{(\mathrm{PS})}_{\mu\nu}\frac{x^a}{r},$$

- It is amusing that the auxillary field configuration is given by

$$F^a_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijm}x_m x_a}{r^4}, \qquad F^a_{tw} = 0.$$

This is the Wu-Yang monopole if we take $c\beta = -\frac{2}{\pi}$.

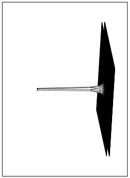
- The BPS equation can be solved with

$$\phi^a = -(u + \frac{\beta}{2\rho^2})\frac{x^a}{r},$$

- The transverse distance $|\phi|$ defined by $|\phi|^2 = \phi^a\phi^a$ gives

$$|\phi| = |u + \frac{\beta}{2\rho^2}|.$$

- This describes a system of M5-branes with a spike at $\rho = 0$ and level off to u as $\rho \to \infty$. Hence the physical interpretation of our self-dual string is that two M5-branes are separating by a distance u and with an M2-brane ending on them.



- Asymptotic $U(1)$ B-field is $\mathcal{B}_{\mu\nu} \equiv \hat{\phi}^a_\infty B^a_{\mu\nu} = \pm B^{(\mathrm{PS})}_{\mu\nu}$ and we obtain

$$P = Q = -\frac{4\pi}{|c|}.$$

Charge quantization

$$PQ' + QP' = 2\pi Z$$

fixes

$$c = \pm 4\sqrt{\pi}$$

- One may generalize the above to a system of N_5 coincident M5-branes with a spike with N_2 self-dual strings.
- We find the spike

$$|\phi| = u + \frac{N_2}{N_5}\frac{1}{\rho^2}.$$

The N_2, N_5 dependence agree precisely with the supergravity solution for intersecting M2-M5 branes. (Niarchos, Siampos)

Outline

I. We have constructed a non-abelian action of tensor fields that we propose to be the bosonic theory describing the gauge sector of coincident M5-branes in flat space.

II. We have also constructed the non-abelian string solutions of the theory. By including a scalar field, we argue how the solution can be promoted to become a solution of the (2,0) theory.
This provides dynamical support to our proposed theory.

Further questions

- Supersymmetry: (2,0)? (1,0)?
  Scalar potential and BPS equation?
- Covariant PST extension of our model?
- Other solutions? M-wave (in progress with Isono)
- Classical integrability? (as compared to 4d self dual YM equation)
- Connection with other proposals? (such as deconstruction approach of Arkani-Hamad etal, instantonic quantum mechanics of Aharony etal, or the 5d SYM proposal of Douglas and Lambert etal?)
  Where is the *B*-field in these descriptions? (similar to the problem of extracting the gravity field in the BFSS matrix model?)